Chapter 11. Mensuration

Question 1

The diagonal of a quadrilateral is 30*m* in length and the length of the perpendiculars to it from the opposite vertices are 6.8 *m* and 9.6*m*. Find the area of the quadrilateral.

Solution:

ABCD be the given quadrilateral BE ⊥ AC and DF ⊥ AC.

Let AC = 30*m*, BE = 6.8*m*, DF = 9.6*m*.

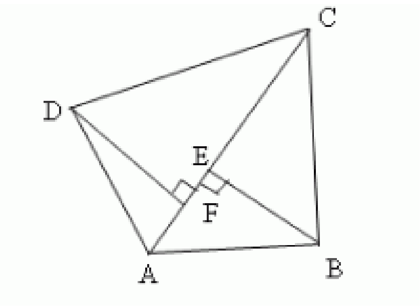
Area of quadrilateral ABCD

$= \text{area of } \Delta ABC + \text{area of } \Delta ACD$

$= \frac{1}{2} \times AC \times BC + \frac{1}{2} \times AC \times DF$

$= \frac{1}{2} \times 30 \times 6.8 + \frac{1}{2} \times 30 \times 9.6$

$= (102 + 144) m^2 = 246 m^2$



Question 2

Find the area of a rhombus the lengths of whose diagonals are 36*cm* and 22.5*cm*.

Solution:

The area of the rhombus $= \frac{1}{2} \times \text{Product of diagonals}$

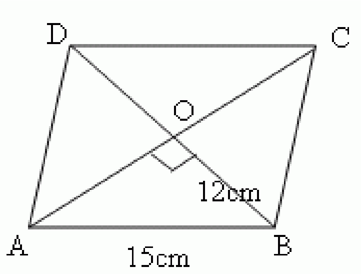
$= \frac{1}{2} \times 36 \times 22.5$

$= 405 cm^2$

Question 3

Find the area of a rhombus in which each side is equal to 15*cm* and one of whose diagonals is 24*cm*.

Solution:



Let ABCD be the rhombus

AB = 15*cm*, BO = 12*cm*.

Since AOB is right angled at O.

By pythagoras them,

$$AO = \sqrt{(AB)^2 - (OB)^2}$$
$$= \sqrt{15^2 - 12^2}$$
$$= \sqrt{225 - 144}$$
$$= \sqrt{81}$$
$$= 9cm.$$

Therefore the two diagonals are 18 *cm* and 24 *cm*.

The area of rhombus $ABCD = \frac{1}{2} \times AC \times BD = \frac{1}{2} \times 18 \times 24 = 216cm^2$

Question 4

Find the area of a trapezium whose parallel sides are 57 *cm* and 39 *cm* and the distance between them is 28 *cm*.

Solution:

The area of the trapezium $= \frac{1}{2} \times (a + b)h$

$$= \frac{1}{2}(57 + 39) \times 28 = 1344cm^2$$

Question 5

The area of trapezium is 352 cm^2. The distance between the parallel sides is 16cm. If one of the parallel sides is 25 cm. Find the other.

Solution:

Area of a trapezium = 352cm^2

$$\frac{1}{2}\times h\times(a+b)=352cm^2$$

$$\frac{1}{2}\times 16\times(25+b)=352cm^2$$

$$8(25+b)=352$$

$$200+8b=352$$

$$8b=352-200=152$$

$$b=\frac{152}{8}=19cm.$$

Question 6

The parallel sides of a trapezium are 25cm and 13cm. Its nonparallel sides are equal each being 10cm. Find the area of the trapezium.

Solution:

AB = 25cm, DC = 13 cm, BC = 10 cm.

AD =10cm

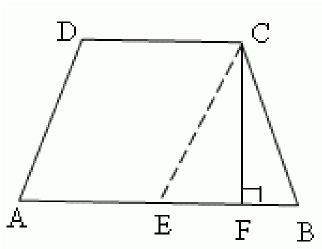
EB = AB - AE = AB - DC

= 25 - 13 = 12cm

CE = AD = 10cm. AE = DC = 13cm

In Δ EBC, CE = BC = 10cm

CF ⊥ AB, F is the midpoint of EB.



$$EF=\frac{1}{2}\times EB=6cm$$

$$CF=\sqrt{CE^2-EF^2}$$

$$=\sqrt{10^2-6^2}$$

$$=\sqrt{64}=8cm$$

Area of trapezium = area of parallelogram AECD + area of Δ CEB

$$=AE\times CF+\frac{1}{2}\times EB\times CF$$

$$=13\times 8+\frac{1}{2}\times 12\times 8$$

$$=152cm^2$$

Question 7

Find the volume, the total surface area and the lateral surface of a cuboid which is 8*m* long, 6*m* broad and 3.5*m* high.

Solution:

The volume of cuboid $= (l \times b \times h) m^3$

$= (8 \times 6 \times 3.5) m^3 = 168 m^3$

The total surface area of the cuboid $= 2(lb + bh + lh) m^2$

$= 2(8 \times 6 + 6 \times 3.5 + 8 \times 3.5) m^2$

$= 194 m^2$

The lateral surface area of the cuboid = $[2(l + b) \times h] m^2$

$= [2(8 + 6) \times 3.5]$

$= 98 m^2$

Question 8

Find the volume of a cube whose total surface area is 486*cm*^{2}.

Solution:

Total surface area = $6a^2$

$6a^2 = 486$

$a^2 = \frac{486}{6} = 81$

$a = \sqrt{81} = 9 cm$

Length of each edge = 9*cm*.

Volume of the cube = $9 \times 9 \times 9 = 729 cm^3$.

Question 9

How many bricks will be required for a wall which is 8m long 6m high and 22.5*cm* thick if each brick measures 25*cm* × 11.25 *cm* × 6*cm*?

Solution:

The volume of the wall $= (800 \times 600 \times 22.5) cm^3$

Volume of 1 brick $= (25 \times 11.25 \times 6) cm^3$

$$\text{Number of bricks required} = \frac{\text{Volume of wall}}{\text{Volume of 1brick}}$$

$$= \frac{800 \times 600 \times 22.5}{25 \times 11.25 \times 6} = 6400$$

Question 10

An open rectangular cistern when measured outside is 1.35 *m* long 1.08*m* broad and 90*cm* deep and is made of iron which is 2.5*cm* thick. Find the capacity of the cistern and the volume of the iron used.

Solution:

The external dimensions of the cistern are

Length = 135 *cm*, breadth = 108*cm*, depth = 90 *cm*.

External volume = $(135\times108\times90)cm^3 = 1312200cm^3$

The internal dimensions of the cistern are

Length = (135 - 5) *cm* = 130 cm, breadth = (108 - 5) *cm* = 103*cm*

Height = (90 - 2.5) *cm* = 87.5 *cm*

The capacity of the cistern = internal volume of the cistern

$= (130\times103\times87.5)cm^3 = 1171625cm^3$

Volume of iron = external volume - internal volume

$= (1312200-1171625)cm^3$

$= 140575cm^3$

Question 11

A field is 80*m* long and 50*m* broad. In one corner of the field, a pit which is 10*m* long, 7.5 *m* broad and 8*m* deep has been dug out. The earth taken out of it is evenly spread over the remaining part of the field. Find the rise in the level of the field.

Solution:

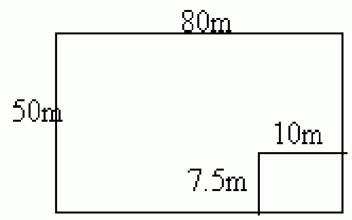
The area of the field $= (80\times50)m^2 = 4000m^2$

The area of the pit $= (10\times7.5)m^2 = 75m^2$

The area over which the earth is spread out

$= (4000-75)m^2 = 3925m^2$

The volume of earth dug out $= (10\times7.5\times8)m^3 = 600m^3$



$\therefore$ The rise in level

$= \frac{\text{volume}}{\text{area}} = \frac{600}{3925}m = \frac{600\times100}{3925} = 15.3cm$.

Question 12

Find the volume of a cylinder which is 18*cm* in height and radius 10.5 *cm*.

Solution:

$r = 10.5cm \quad h = 18cm$

The volume of the cylinder $= \pi r^2 h$

$= \left(\frac{22}{7} \times 10.5 \times 10.5 \times 18\right) cm^3$

$= 6237 cm^3$

Question 13

How many cubic metres of earth must be dug out to since a well is 16*m* deep and which has a radius of 3.5*m*? If the earth taken out is spread over a rectangular plot of dimensions $25m \times 16m$ what is the height of the platform so formed?

Solution:

The volume of the earth dug out $= \pi r^2 h$

$= \left(\frac{22}{7} \times 3.5 \times 3.5 \times 16\right) m^3$

$= 616 m^3$

The area of the given plot $= (25 \times 16) m^2 = 400 m^2$

The volume of the platform formed = the volume of the earth dug out

$= 616 m^3$

The height of the platform $= \frac{\text{volume}}{\text{area}} = \frac{616}{400} = 1.54m$

The height of the platform = 1.54*m*.

Question 14

An iron pipe is 21*cm* long and its exterior diameter is 8*cm*. If the thickness of the pipe is 1cm and iron weighs 8*g/cm*^{3}. Find the weight of the pipe.

Solution:

The external radius of the pipe = 4*cm*

The internal radius of the pipe = (4 - 1) cm = 3*cm*

The external volume = $\left(\frac{22}{7}\times4\times4\times21\right)cm^3 = 1056cm^3$

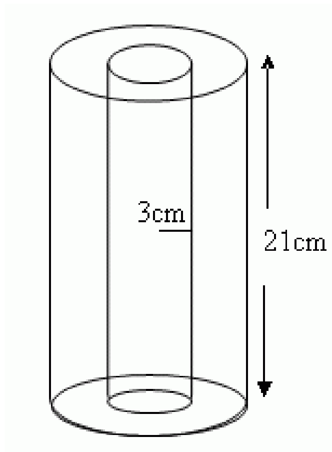
The internal volume = $\left(\frac{22}{7}\times3\times3\times21\right)cm^3 = 594cm^3$

The volume of the metal = external volume - internal volume

$= (1056-594)cm^3 = 462cm^3$

The weight of the pipe = $(462\times8)g$

$= \frac{462\times8}{1000}kg = 3.696kg$



Question 15

A closed metallic cylindrical box is 1.25*m* high and it has a base whose radius is 35*cm*. If the sheet of metal costs Rs. 80 per *m*^{2}. Find the cost of the material used in the box. Find the capacity of box in litres.

Solution:

$h = 1.25m \quad r = \frac{35}{100}m = 0.35m$

The area of metal used = total surface area of the box

$= 2\pi r(h+r)$

$= 2\times\frac{22}{7}\times0.35(1.25+0.35)m^2$

$= \left(2\times\frac{22}{7}\times0.35\times1.6\right)m^2 = 3.52m^2$

The cost of material used = $Rs(3.52\times80) = Rs.281.60$

$h = 125cm = 12.5dm \quad r = 35cm = 3.5dm$

The capacity of the box = volume of the box

$= \pi r^2h$ cu. units .

$= \frac{22}{7}\times3.5\times3.5\times12.5dm^3$

Question 16

A rectangular piece of paper of dimensions 22*cm* by 12*cm*. is rolled along its length to form a cylinder. Find the volume of the cylinder so formed.

Solution:

Length of paper = Height of the cylinder = 12*cm*

Circumference of its base = 22 *cm*

$$2\pi r = 22$$

$$r = 22 \times \frac{7}{22 \times 2} = \frac{7}{2} cm,$$

Volume of the cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12 cm^3$$

$$= 462 cm^3$$

Question 17

PQRS is a quadrilateral in which PQ =4 *cms*, QR = 9.2 *cms*, RS = 8*cm* SP = 6*cm* ∠ PSR = ∠ PQR = 90° . Find its area.

Solution:

Area of right angled $\Delta PQR = \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times 4 \times 9.2$$

$$= 2 \times 9.2 = 18.4 cm^2$$

Area of right angled $\Delta PSR = \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times 8 \times 6 = 4 \times 6 = 24 cm^2$$

Area of quadrilateral $PQRS$ = Area of ΔPQR + Area of ΔPSR

$$= 18.4 + 24 = 42.4 cm^2$$

Question 18

Find the surface area of a chalk box whose length, breadth and height are 16*cm*, 8*cm* and 6*cm* respectively.

Solution:

Chalk box is in the form of cuboid.

$l = 16cm,\ b = 8cm,\ h = 6cm$

Surface area of the cuboid $= 2(lb + bh + lh)$

$= 2(16\times8 + 8\times6 + 16\times6)cm^2$

$= 2(128 + 48 + 96)cm^2 = 544cm^2$

$\therefore$ The area of the chalk box is 544 cm^2.

Question 19

The dimensions of a cuboid are in the ratio of 1:2:3 and its total surface area is 88m^2. Find its dimensions.

Solution:

The dimensions of the cuboid are in the ratio 1:2:3

Let the dimensions be $x, 2x, 3x$ in metres

Surface area $= 88m^2$

$$2(x\times2x + 2x\times3x + x\times3x) = 88$$

$$2(2x^2 + 6x^2 + 3x^2) = 88$$

$$2\times11x^2 = 88$$

$$x^2 = \frac{88}{22} = 4$$

$$x = 2m$$

$2x = 2\times2 = 4m,\ 3x = 3\times2 = 6m$

$\therefore$ The dimensions are 2 m, 4 m and 6 m.

Question 20

The length of a hall is 20*m* and width *16m*. The sum of the areas of the floor and the flat roof is equal to the sum of the areas of the four walls. Find the height and volume of the hall?

Solution:

Let '*h*' be the height of the wall.

Sum of areas of four walls $= 2(l+b)hm^2 = 2(20+16)hm^2 = 72hm^2$

Sum of the areas of the floor and the flat roof = $20\times16+20\times16 = 640m^2$

Given that the sum of the areas of four walls is equal to the sum of the areas of the floor and roof

$$72h = 640$$

$$h = \frac{640}{72} = \frac{80}{9}m = 8.88m$$

Volume of the hall $= 20\times16\times\frac{80}{9}m^3 = \frac{25600}{9}m^3 = 2844.4m^3$.